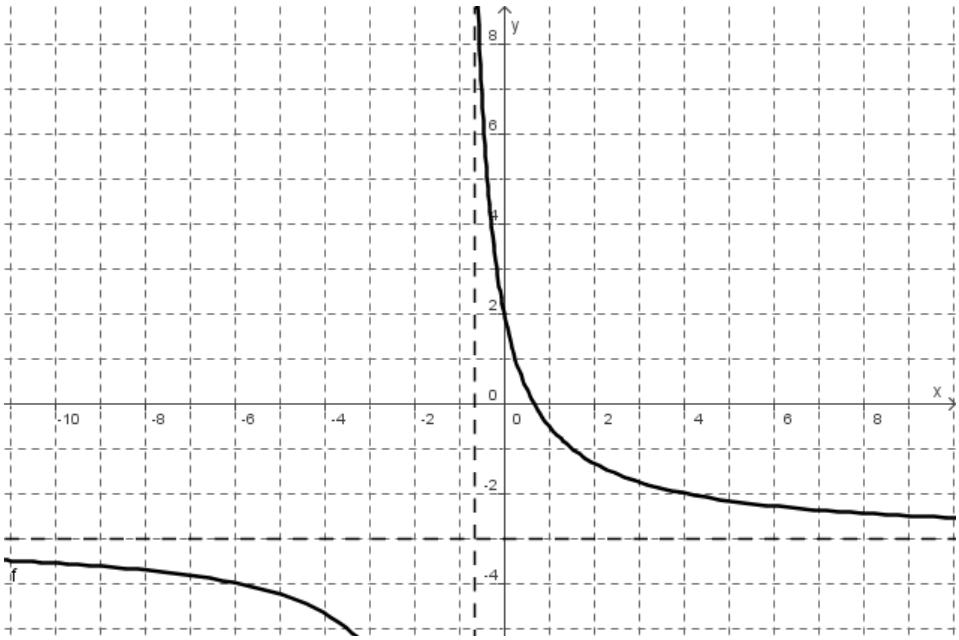
Klasse	Art	Schwierigkeit	Thema	S. 21
11	**Üben**	**X**	**Gebrochen-rationale Funktionen 3**	**3**

a) $f(x) = \frac{2-3x}{x+1}$ $\quad D_f = \mathbb{R} \setminus \{-1\}$

Symmetrie? $f(-x) = \frac{2-3(-x)}{-x+1} = \frac{2+3x}{-x+1} \neq \begin{cases} f(x) \\ -f(x) \end{cases}$ $\Rightarrow$ keine Symmetrie

Asymptoten : x = -1 ; y = - 3

Nullstelle $(\frac{2}{3} / 0)$



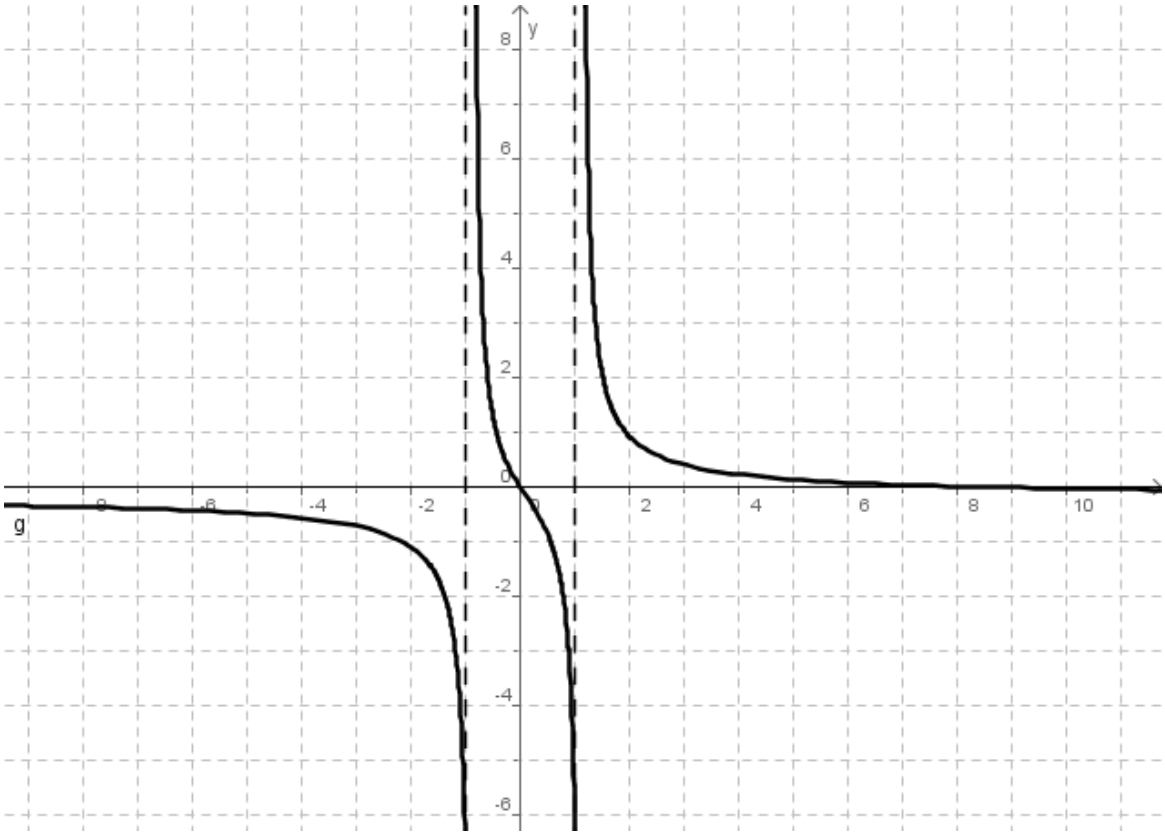
b) $g(x) = \frac{1{,}5x}{x^2 - 1}$ $\quad D_f = \mathbb{R} \setminus \{-1;1\}$

Symmetrie?

$f(-x) = \frac{1{,}5(-x)}{(-x)^2 - 1} = \frac{-1{,}5x}{x^2 - 1} = -f(x) \Rightarrow$ Punktsymmetrie zum Ursprung

Asymptoten : x = -1 ; x = 1; y = 0

Nullstelle $(0 / 0)$



c) $f(x) = \frac{1}{2}x - 1 - \frac{2}{3x+2}$ $\quad D_f = \mathbb{R} \setminus \{-\frac{2}{3}\}$

Symmetrie?

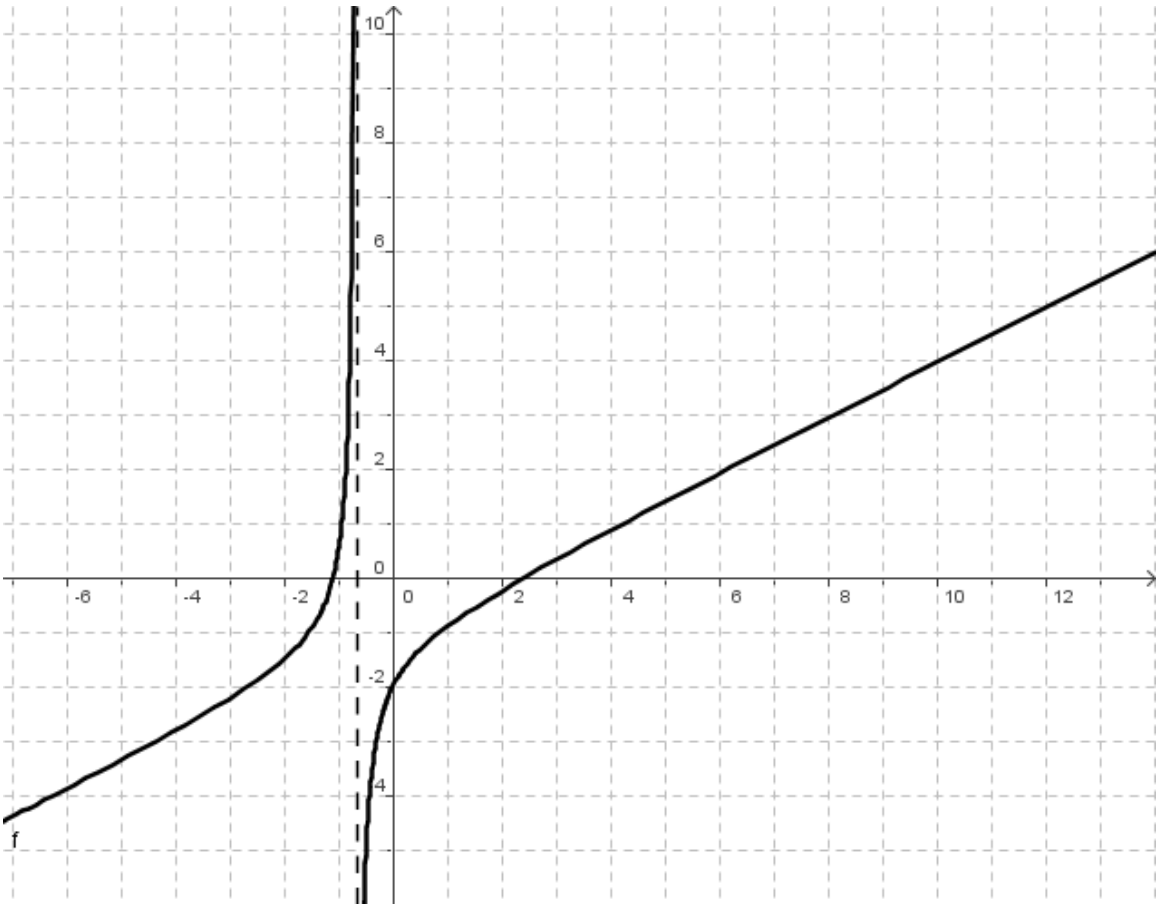
$f(-x) = \frac{1}{2}(-x) - 1 - \frac{2}{3(-x)+2} = -\frac{1}{2}x - 1 - \frac{2}{-3x+2} \neq \begin{cases} f(x) \\ -f(x) \end{cases}$ keine Symmetrie

Asymptoten : $x = -\frac{2}{3}$; $y = \frac{1}{2}x - 1$

Nullstelle ?

$$f(x) = \frac{1}{2}x - 1 - \frac{2}{3x+2} = \frac{(\frac{1}{2}x - 1)(3x+2) - 2}{3x+2} = \frac{\frac{3}{2}x^2 + x - 3x - 2 - 2}{3x+2} = \frac{\frac{3}{2}x^2 - 2x - 4}{3x+2}$$

$f(x) = 0$, wenn $\frac{3}{2}x^2 - 2x - 4 = 0$ $\quad x = \frac{2 \pm 2\sqrt{7}}{3}$



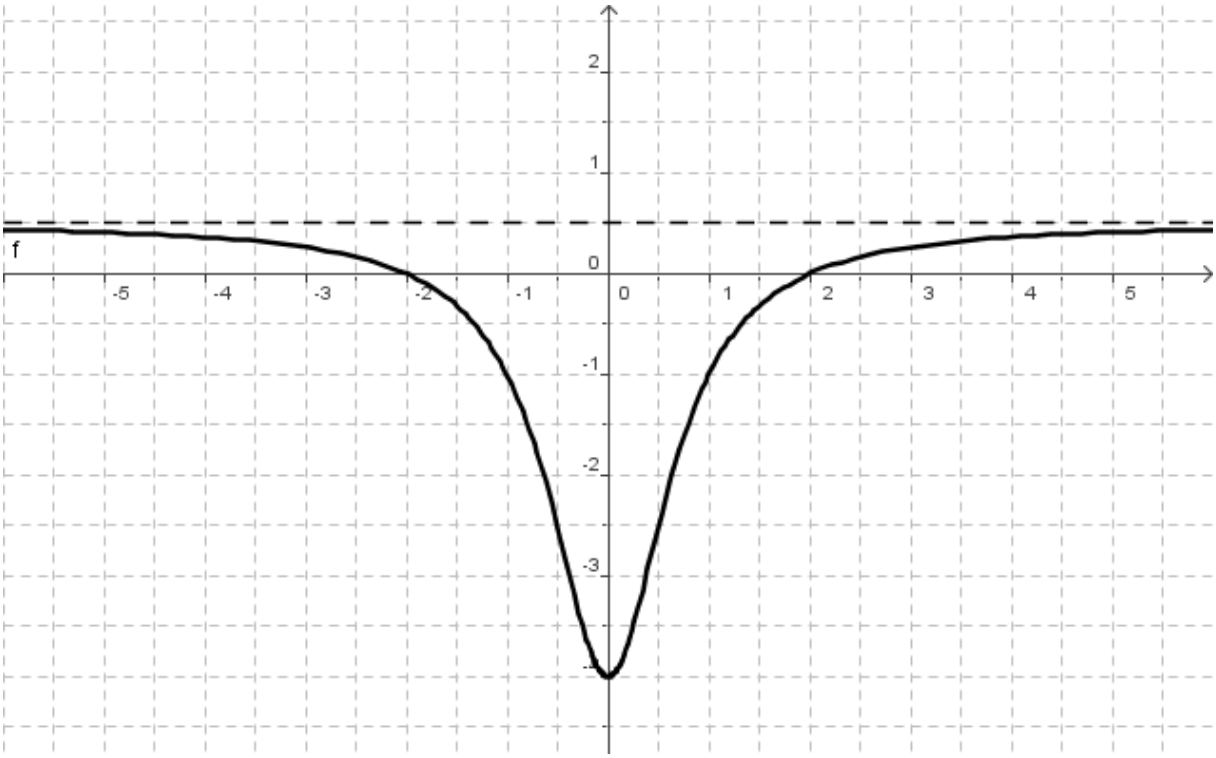
d) $f(x) = \frac{x^2 - 4}{2x^2 + 1}$

Definitionsmenge: $D_f = \mathbb{R}$

Symmetrie? $f(-x) = \frac{(-x)^2 - 4}{2(-x)^2 + 1} = \frac{x^2 - 4}{2x^2 + 1} = f(x) \Rightarrow$ Achsensymmetrie zur y-Achse

Asymptoten : $y = \frac{1}{2}$

Nullstellen: $N_1(-2/0)$; $N_2(2/0)$



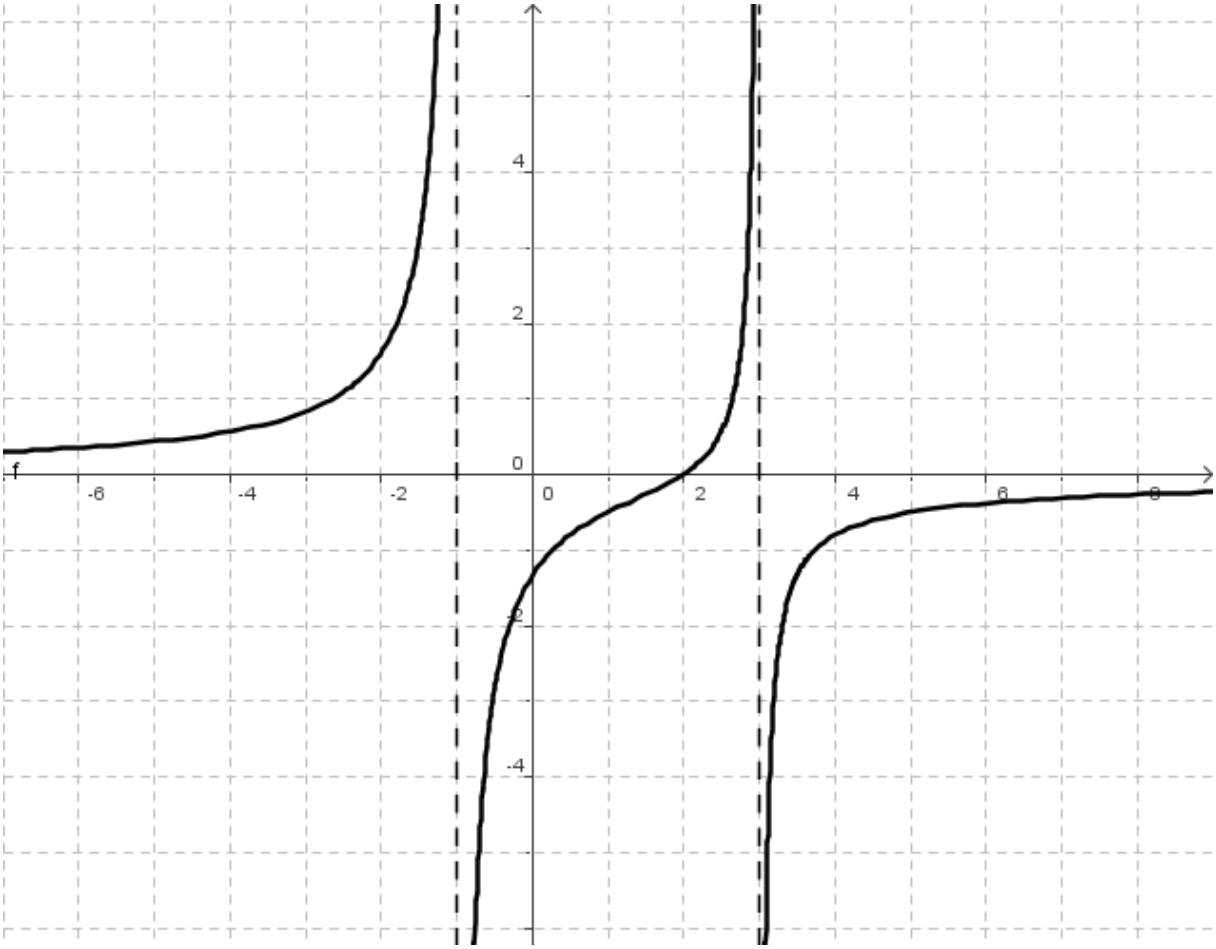
e) $g(x) = \dfrac{4-2x}{x^2-2x-3}$

Definitionsmenge: $D_f = \mathbb{R} \setminus \{-1\,;\,3\}$

Symmetrie? $g(-x) = \dfrac{4-2(-x)}{(-x)^2-2(-x)-3} = \dfrac{4+2x}{x^2+2x-3} \neq \begin{cases} g(x) \\ -g(x) \end{cases}$ keine Symmetrie

Asymptoten : $y = 0$; $x = -1$; $x = 3$

Nullstellen: N(2/0)



f) $g(x) = \dfrac{2x^2 - 1}{x - 2}$

Definitionsmenge: $D_f = \mathbb{R} \setminus \{-2\}$

Symmetrie? $g(-x) = \dfrac{2(-x)^2 - 1}{-x - 2} = \dfrac{2x^2 - 1}{-x - 2} \neq \begin{cases} g(x) \\ -g(x) \end{cases}$ keine Symmetrie

Asymptoten : $g(x) = \dfrac{2x^2 - 1}{x - 2} = 2x + 4 + \dfrac{7}{x - 2}$ $\Rightarrow$ Schiefe Asympt.: $y = 2x + 4$;

senkrechte Asympt.: $x = 2$

Nullstellen: bei $x_{1/2} = \pm\sqrt{0{,}5}$